1.8 Transformations of Functions

$$y = a f[k(x-d)] + c$$

f = the function: squareroot, absolute value...

on a graph, a function can be transformed in 4 ways:

Horizontally, vertically, shifted, stretched.

Horizontal: x values are affected

Vertical: y values are affected

Shift: moves - add or subtract

Stretch/compression: changes the look of graph, by making it skinny or fatter.

$$y = a f[k(x-d)] + c$$

Horizontal values are inside the fn. do the opposite.

Vertical values are outside the fn.

a = vertical stretch (v.str) : $y(a)$

k = horizontal stretch (h.str) : $x\left(\frac{1}{k}\right)$

d = horizontal shift (h.sh) : $x\left(\frac{1}{k}\right) - d$

opposite of d

c = vertical shift (v.sh) : $y(a) + c$

Handout:

a) $y = \underbrace{-2}_{a} f(x) \underbrace{+ 3}_{c}$

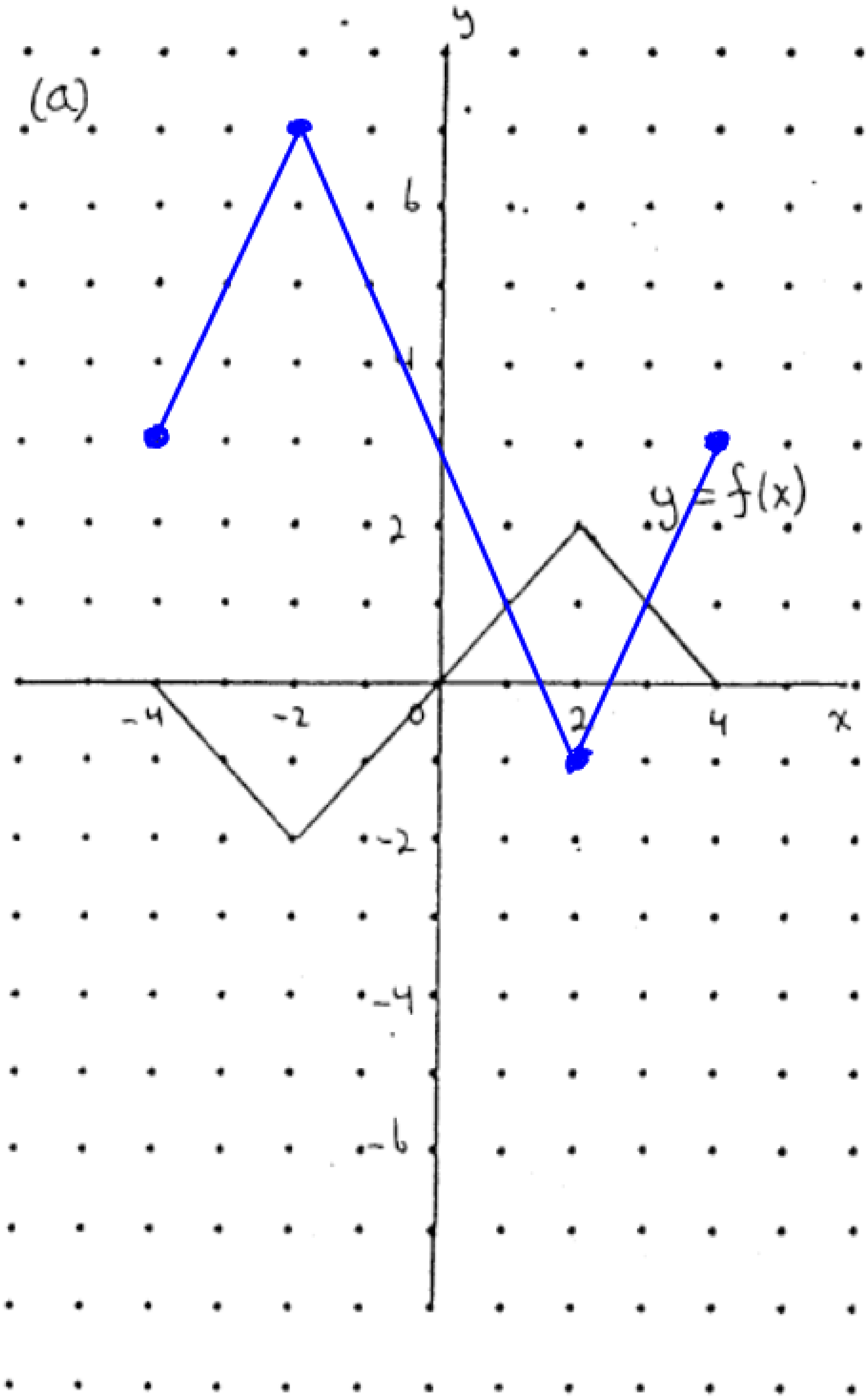
① V. Str is -2

② V. Sh is $+3$

x	y
-4	0
-2	-2
2	2
4	0

x	$-2y+3$
-4	3
-2	7
2	-1
4	3

Graph these!



(a)
y
x
6
4
2
-2
-4
-6
-4
-2
0
2
4
y = f(x)

b) $y = \underset{a}{\underline{\frac{1}{2}}} f(x \underset{d}{\underline{-2}}) \underset{c}{\underline{-6}}$

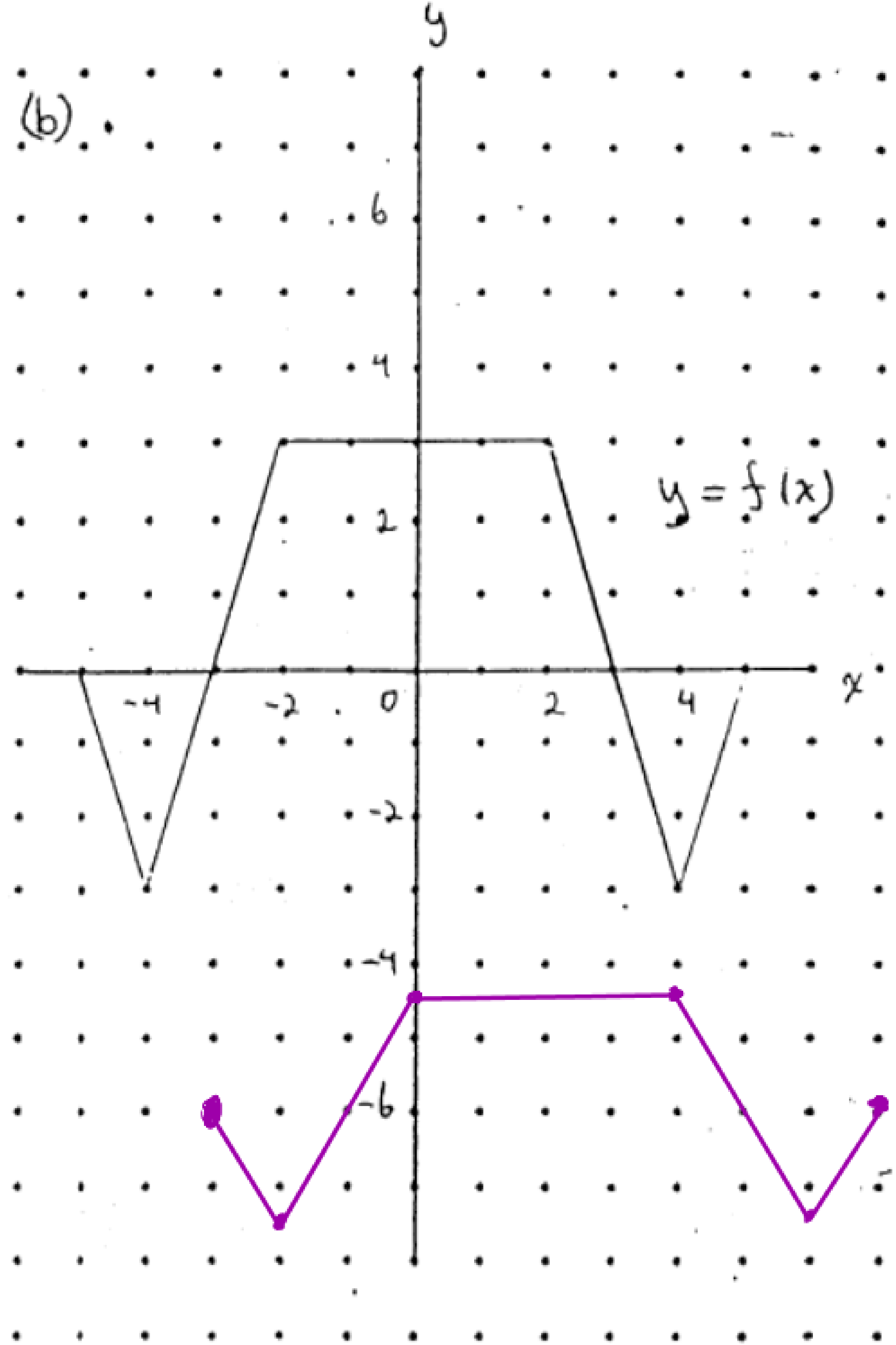
① V. Str of $\frac{1}{2}$ ② H. Sh of $+2$

③ V. Sh of -6

x	y
-5	0
-4	-3
-2	3
2	3
4	-3
5	0

$x+2$	$\frac{1}{2}y-6$
-3	-6
-2	-7.5
0	-4.5
4	-4.5
6	-7.5
7	-6

(b)



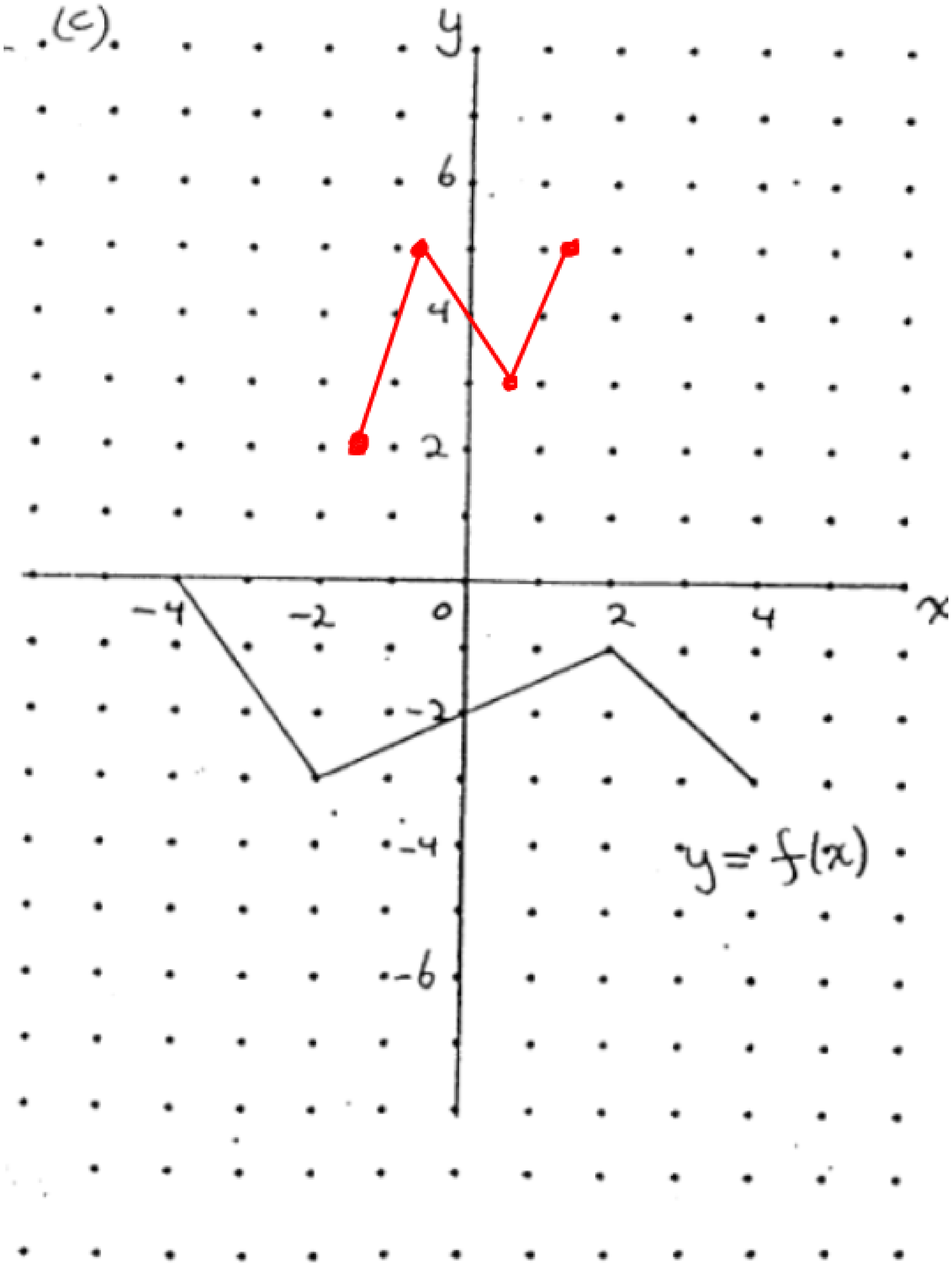
c) $y = \underset{a}{\underline{-}} f(\underset{k}{\underline{3}}x) + \underset{c}{\underline{2}}$

① V. Str of -1 ② H. Str of $\frac{1}{3}$ ③ V. Sh of $+2$

x	y
-4	0
-2	-3
2	-1
4	-3

$\frac{1}{3}x$	$-y+2$
$-4/3$	2
$-2/3$	5
$2/3$	3
$4/3$	5

(c)



New example:

$$f(x) = \underset{a}{\underline{-2}}\sqrt{\underset{k}{\underline{-\tfrac{1}{3}}}(x + \underset{d}{\underline{4}})} + \underset{c}{\underline{5}}$$

$\boxed{\text{Parent fn} \quad f(x) = \sqrt{x}}$

(1) V. str of -2

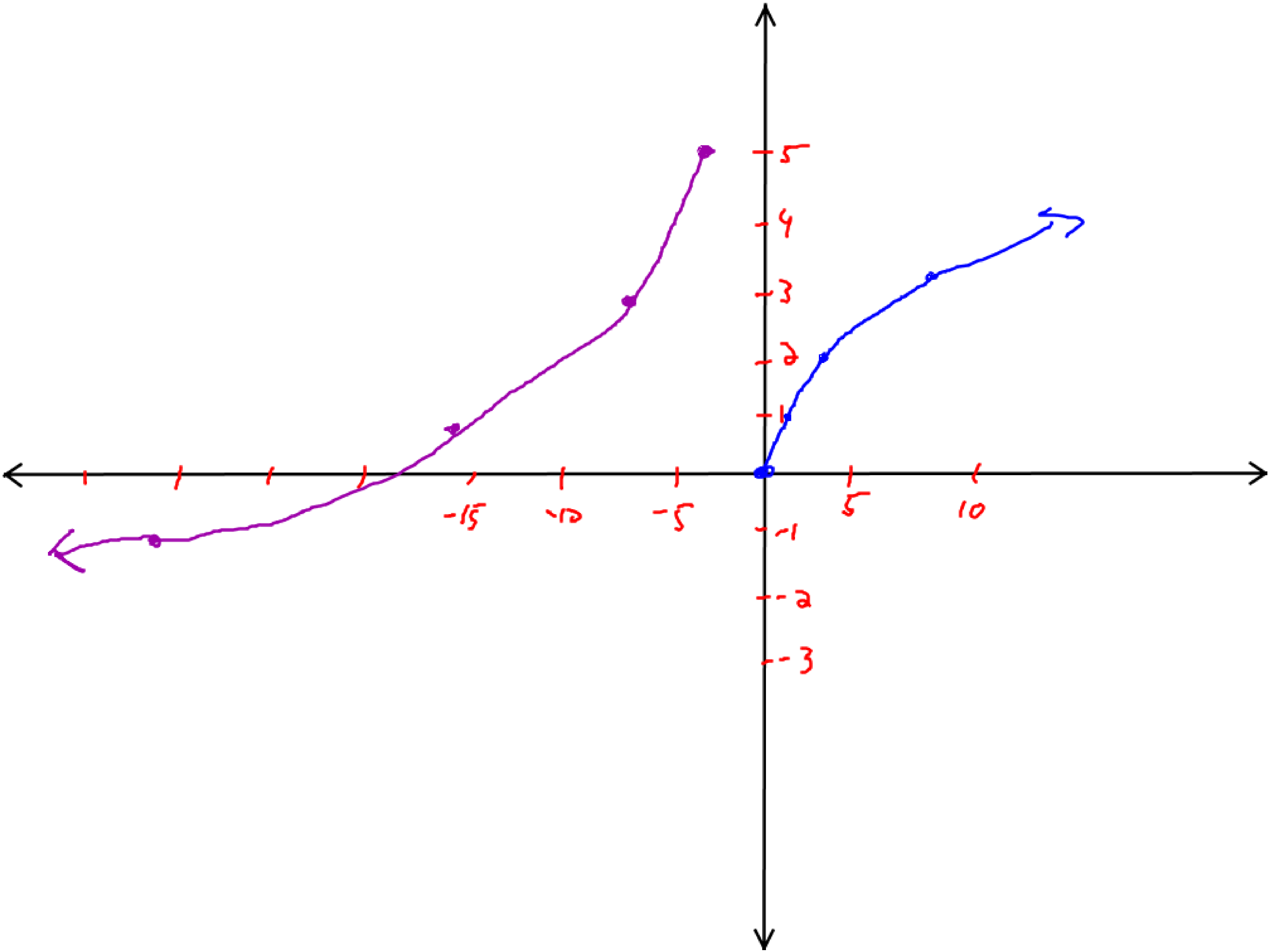
(2) H. str of -3

(3) V. Sh of 5

(4) H. Sh of -4

x	y
0	0
1	1
4	2
9	3

$-3x-4$	$-2y+5$
-4	5
-7	3
-16	1
-31	-1



5
4
3
2
1
-1
-2
-3
-15
-10
-5
5
10

Homework:

Absolute values.

$$f(x) = 4\left|2(x+8)\right| + 3$$